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## Sheet (2)

1. A hypothetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field ( $E_\theta$ ) is measured to be 5 V/m. Find the
  - (a) Power density ( $W_{\text{rad}}$ )
  - (b) Power radiated ( $P_{\text{rad}}$ )
2. The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of
$$U = B_0 \cos^3 \theta$$
 (watts/unit solid angle) ( $0 \leq \theta \leq \pi/2$ ,  $0 \leq \phi \leq 2\pi$ )  
Find the
  - (a) Maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.
  - (b) Exact and approximate beam solid angle  $\Omega_A$ .
  - (c) Directivity, exact and approximate, of the antenna (dimensionless and in dB).
  - (d) Gain, exact and approximate, of the antenna (dimensionless and in dB).
3. In target-search ground-mapping radars it is desirable to have echo power received from a target, of constant cross section, to be independent of its range. For one such application, the desirable radiation intensity of the antenna is given by

$$U(\theta, \phi) = \left\{ \begin{array}{ll} 1 & 0^\circ \leq \theta < 20^\circ \\ 0.342 \csc(\theta) & 20^\circ \leq \theta < 60^\circ \\ 0 & 60^\circ \leq \theta \leq 180^\circ \end{array} \right\} 0^\circ \leq \phi \leq 360^\circ$$

Find the directivity (in dB) using the exact formula.

4. The normalized radiation intensity of a given antenna is given by
  - (a)  $U = \sin \theta \sin \phi$ , (b)  $U = \sin \theta \sin^2 \phi$ , (c)  $U = \sin^2 \theta \sin^3 \phi$The intensity exists only in the  $0 \leq \theta \leq \pi$ ,  $0 \leq \phi \leq \pi$  region, and it is zero elsewhere. Find the
  - (a) Exact directivity (dimensionless and in dB).
  - (b) Azimuthal and elevation plane half-power beam widths (in degrees).



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5. Find the directivity (dimensionless and in dB) for the antenna of Problem 4 using Kraus' approximate formula.

6. The normalized radiation intensity of an antenna is rotationally symmetric in  $\phi$ , and it is represented by

$$U = \begin{cases} 1 & 0^\circ \leq \theta < 30^\circ \\ 0.5 & 30^\circ \leq \theta < 60^\circ \\ 0.1 & 60^\circ \leq \theta < 90^\circ \\ 0 & 90^\circ \leq \theta \leq 180^\circ \end{cases}$$

(a) What is the directivity (above isotropic) of the antenna (in dB)?

7. The radiation intensity of an antenna is given by  $U(\theta, \phi) = \cos^4 \theta \sin^2 \phi$ , for  $0 \leq \theta \leq \pi/2$  and  $0 \leq \phi \leq 2\pi$  (i.e., in the upper half-space). It is zero in the lower half-space.

Find the

(a) Exact directivity (dimensionless and in dB)

(b) Elevation plane half-power beam width (in degrees).

8. The far-zone electric-field intensity (array factor) of an end-fire two-element array antenna, placed along the z-axis and radiating into free-space, is given by

$$E = \cos \left[ \frac{\pi}{4} (\cos \theta - 1) \right] \frac{e^{-jkr}}{r}, \quad 0 \leq \theta \leq \pi$$

Find the directivity using Kraus' approximate formula

9. The normalized far-zone field pattern of an antenna is given by

$$E = \begin{cases} (\sin \theta \cos^2 \phi)^{1/2} & 0 \leq \theta \leq \pi \text{ and } 0 \leq \phi \leq \pi/2, 3\pi/2 \leq \phi \leq 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Find the directivity using

(a) The exact expression

(b) Kraus' approximate formula



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10. A uniform plane wave, of is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle  $\tau$  (in degrees) when
- (a)  $E_x = E_y, \Delta\phi = \phi_y - \phi_x = 0$
  - (b)  $E_x \neq E_y, \Delta\phi = \phi_y - \phi_x = 0$
  - (c)  $E_x = E_y, \Delta\phi = \phi_y - \phi_x = \pi/2$
  - (d)  $E_x = E_y, \Delta\phi = \phi_y - \phi_x = -\pi/2$
  - (e)  $E_x = E_y, \Delta\phi = \phi_y - \phi_x = \pi/4$
  - (f)  $E_x = E_y, \Delta\phi = \phi_y - \phi_x = -\pi/4$
  - (g)  $E_x = 0.5E_y, \Delta\phi = \phi_y - \phi_x = \pi/2$
  - (h)  $E_x = 0.5E_y, \Delta\phi = \phi_y - \phi_x = -\pi/2$
- In all cases, justify the answer.

### (REPORT)

1. The normalized radiation intensity of an antenna is symmetric, and it can be approximated by

$$U(\theta) = \begin{cases} 1 & 0^\circ \leq \theta < 30^\circ \\ \frac{\cos(\theta)}{0.866} & 30^\circ \leq \theta < 90^\circ \\ 0 & 90^\circ \leq \theta \leq 180^\circ \end{cases}$$

And it is independent of  $\phi$ . Find the

- (a) Exact directivity by integrating the function
- (b) Approximate directivity using Kraus' formula.

2. Repeat Problem 8 when

$$E = \cos \left[ \frac{\pi}{4} (\cos \theta + 1) \right] \frac{e^{-jkr}}{r}, \quad 0 \leq \theta \leq \pi$$

*Good Luck*